LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2025**

UMT 1502 – CALCULUS

Date: 26-04-2025 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A - K1 (CO1)	
	Answer ALL the Questions	**(10 x 1 = 10)**
1.	**Answer the following**	
a)	State Leibnitz Theorem to find n^{th} derivative of a product of two functions.	
b)	Determine the slope of the tangent line to the polar curve $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{3}$.	
c)	Find $\int xe^x dx$.	
d)	Rewrite $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$ by changing its order.	
e)	State any two properties of beta function.	
2.	**Fill in the blanks**	
a)	The n^{th} derivative of $(3x+5)^m$ is ……	
b)	The formula to find radius of curvature of $y = f(x)$ is …….	
c)	The change of order of $\int_0^2 \int_{x^2}^{2x} f(x,y)\, dy\, dx$ is ……..	
d)	The integral of $\frac{1}{\sqrt{2x}}$ with respect to x is …….	
e)	The value of $\Gamma\left(\frac{5}{2}\right)$ is ……	
	SECTION A - K2 (CO1)	
	Answer ALL the Questions	**(10 x 1 = 10)**
3.	**Choose the correct answer for the following**	
a)	A function $f(x,y)$ attains a maximum value if $rt - s^2 > 0$ and r (i) > 0 (ii) < 0 (iii) $= 0$ (iv) None of the above	
b)	The subtangent on any point on the parabola $y^2 = 4ax$ is (i) $2a$ (ii) $2x$ (iii) x (iv) a	
c)	If f is an even function, then $\int_{-a}^{a} f(x)dx$ is _____ (i) $2\int_{-a}^{a} f(x)dx$ (ii) $2\int_{0}^{a} f(x)dx$ (iii) $2\int_{a}^{0} f(x)dx$ (iv) 0	
d)	Reduction formula for $\int_0^{\pi/2} \sin^n x\, dx$ is (i) $I_n = \frac{(n+1)}{n} I_{n+1}$	

	(ii) $I_n = \frac{(n-1)}{n} I_{n-1}$ (iii) $I_n = \frac{(n-1)}{n} I_{n-2}$ (iv) $I_n = \frac{(n-2)}{n} I_{n-2}$
e)	The definite integral of $x^3(1-x)$ from 0 to 1 is …….. (i) $-\frac{1}{20}$ (ii) 1 (iii) $\frac{1}{20}$ (iv) 20
4.	**State True or False**
a)	The method of Lagranges multipliers is a technique applied to determine the local maxima and minima of a function of the form $f(x,y,z)$ subject to equality constraints.
b)	The evolute of a curve is the locus of center of curvature.
c)	The tangent to the evolute at any point on it is the normal to the curve at the corresponding point on the curve.
d)	$\int_0^a f(x)dx = \int_0^{-a} f(a-x)dx.$
e)	Recurrence formula for gamma functions is true only if n is greater than or equal to zero.
	SECTION B - K3 (CO2)
	Answer any TWO of the following **(2 x 10 = 20)**
5.	Find the lengths of the subtangent, subnormal, tangent and normal at the point (a,a) on the cissoids $y^2 = \frac{x^3}{2a-x}$.
6.	Determine angle of intersection of the curve $r = a(1+\cos\theta)$ and $r = b(1-cos\theta)$.
7.	Evaluate $\int_0^{\frac{\pi}{2}} \frac{(sinx)^{\frac{3}{2}}}{(sinx)^{\frac{3}{2}}+(cosx)^{\frac{3}{2}}} dx.$
8.	Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dzdydx.$
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 10 = 20)**
9.	Find the coordinates of centre of curvature of $xy = 2$ at $(2,1)$.
10.	Compute the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1,1).
11.	Find the value of the integral $\int\int\int xyz\, dxdydz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.
12.	Prove that $\int_0^{\frac{\pi}{4}} \log(1+tan\theta)\, d\theta = \frac{\pi}{8}\log 2.$
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 20 = 20)**
13.	Defend that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.
14.	Discuss the maxima and minima of the function $2(x^2-y^2) - x^4 + y^4$.
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
15.	If $y = \sin(m\, sin^{-1}x)$, prove that $(1-x^2)y_2 - xy_1 + m^2y = 0$ and show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$
16.	Derive an angle between the radius vector and the tangent and hence find the angle at which the radius vector cuts the curve $\frac{l}{r} = 1 + ecos\,\theta$.

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